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A Comparison of Finite-Element Methods for Solving Flow Past a Sphere

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Three finite-element methods for calculating the drag coefficient for a sphere in steady, laminar flow at low or intermediate Reynolds numbers are compared. The flow equations are solved either for the stream function and the vorticity or for the velocity and pressure, with different boundary conditions applied far from the sphere. It is found that accurate values of the drag coefficient can be obtained on coarser grids using a velocity–pressure formulation, even though a physically realistic boundary condition can be applied closer to the sphere with the stream-function–vorticity formulation. In addition calculated values of the drag coefficient are compared with accepted correlations.

1. INTRODUCTION

Over the years there have been many numerical calculations of the drag on a sphere in steady, laminar flow. Hamielec, Hoffman, and Ross [1] and Le Clair, Hamielec, and Pruppacher [2] solved the stream-function–vorticity equations with a finite-difference method. Dennis and Walker [3] used a semi-analytical formulation whereby the flow variables were expanded as truncated series of Legendre functions in the angular variable, hence reducing the equations to ordinary differential equations in the radial variable. The resulting equations were solved numerically. More recently Sayegh and Gauvin [4] examined the effects of the variation of fluid properties with temperature and Renksizbulut [5] and Renksizbulut and Yuen [6] have studied the problem with heat and mass transfer from the sphere. These authors evaluated their finite-difference numerical methods by solving the isothermal problem. The results of all these calculations vary by about 5 %, reflecting the problems of external flow calculations where the effects of the sphere are significant at distances many times its radius. Finally we have reported some calculations [7, 8], believed to be very accurate, using a finite-element method to solve a stream-function–vorticity formulation ($\phi - \zeta$ formulation) of the flow equations.

In this paper we extend this work by comparing three finite-element calculations of the drag coefficient: one using a velocity–pressure formulation ($\mathbf{u} - p$ formulation); and two, with different conditions on the outer boundary of the computational region far from the sphere, using the stream-function–vorticity for-

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mulation. As the grid is refined and the outer boundary moved further away from the sphere, the computed drag coefficients for these different formulations converge to the actual value in contrasting ways.

The important parameter for describing the flow is the Reynolds number, defined as $\mathrm{Re} = dU\rho/\mu$, where d is the diameter of the sphere, U its velocity relative to the surrounding fluid, and ρ and μ respectively the density and the dynamic viscosity of the fluid. For $\mathrm{Re} \lesssim 130$ the flow is laminar and stable; as Re is increased beyond about 130 a weak long-period oscillation appears in the wake. So we compare the three solution methods for two values of Reynolds number, 1 and 40; flows at these values are typical of results at low and intermediate Reynolds number. Finally, we tabulate values of the drag coefficient for Re in the range 1 to 100, and compare the values with the correlations suggested by Clift, Grace, and Weber [9].

2. Stream-Function–Vorticity Formulation

The Navier–Stokes equations can be formulated in terms either of the stream function and the vorticity or of the velocity and pressure, and we consider these in turn.

It is natural to use spherical polar coordinates (r, θ, ϕ) and, with axisymmetry, the equations for the stream function and vorticity are

$$\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta} = -r\zeta, \tag{1}$$

$$\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}\frac{\partial\zeta}{\partial r} - \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial r}\frac{\partial\zeta}{\partial\theta} - \frac{\zeta}{r^3\sin\theta}\frac{\partial\psi}{\partial\theta} + \frac{\zeta\cot\theta}{r^2\sin\theta}\frac{\partial\psi}{\partial r}$$
$$-\frac{2}{\mathrm{Re}}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\zeta}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\zeta}{\partial\theta} - \frac{\zeta}{r^2\sin^2\theta}\right\} = 0, \tag{2}$$

where ψ is the stream function and ζ is the vorticity. The equations have been made dimensionless by dividing the radial coordinate by the radius of the sphere, and the velocity by the free stream value. These equations are solved in the region given by $1 \leqslant r \leqslant r_\infty$ and $0 \leqslant \theta \leqslant \pi$. For computational purposes we introduce the variable $\xi = \ln(r)$ and treat ξ and θ as the independent variables. This change of variables produces a natural compression, in real space, of the grid lines near the sphere.

Equations (1) and (2) must be supplemented by appropriate boundary conditions. Along the downstream and upstream symmetry axes ψ and ζ are both zero. Around the surface of the sphere $(r = 1)$ the no-slip condition requires that both ψ and $\partial\psi/\partial r$ are zero. It is more difficult to specify the boundary condition on the outer boundary of the computation region $(r = r_\infty)$. Vorticity is created at the surface of the sphere and then diffuses away from the surface and is convected downstream by the flow. So ζ is set to zero on the part of the boundary at $r = r_\infty$ that is the inflow boundary $(\pi/2 \leqslant \theta \leqslant \pi)$, expressing the fact that there is no vorticity in the free stream. On the other part, $0 \leqslant \theta \leqslant \pi/2$, the numerically convenient condition $\partial\zeta/\partial r = 0$ is used, and because of the form of the vorticity equation, any error introduced decays exponentially away from the boundary.

In addition a further condition on the stream function is required. That used in most previous calculations [1–6], and also considered in this comparison, is the free stream condition, which implies that at sufficiently large r_0 the stream function is unaffected by the presence of the sphere. If we write

$$\psi = \psi_{\rm FS} + \psi_p,$$

where $\psi_{\rm FS}$ is the stream function in the absence of the sphere ($\frac{1}{2}r^2 \sin^2 \theta$), and ψ_p is the perturbation due to the presence of the sphere, then the free stream boundary condition is

$$\psi_p = 0 \qquad \text{at} \quad r = r_\infty. \tag{3}$$

A better boundary condition may be developed by noting that at large distances from the sphere and for arbitrary Reynolds number, the perturbed component of the flow has two parts [10]. There is an inflow in the wake region which is associated with the momentum defect, the momentum removed from the free stream which produces the drag on to the sphere. To compensate this inflow there is a uniform radial flow out from the sphere which, at large distances, resembles that from a point source of mass. At sufficiently large distances the entire perturbed flow is radial; this implies

$$\frac{\partial \psi_p}{\partial r} = 0. \tag{4}$$

The condition (4) is much easier to apply in a stream-function–vorticity formulation than in a velocity–pressure formulation. This condition has been used previously by Fornberg [11] in a study of flow past a circular cylinder. It has a more secure physical basis than the free stream boundary condition and we thus expect to be able to apply it closer to the sphere.

The flow equations in the region about the sphere were discretized according to the Galerkin finite-element method, by means of nine-node biquadratic elements. The method is similar to that of Tong [12] and Cliffe and Winters [13] and was implemented using the TGSL finite-element package developed at Harwell [14]. Now since the condition (4) applies to ψ_p rather than to ψ, it is natural to treat ψ_p and ζ as the dependent variables after substituting the free stream function, $\psi_{\rm FS} = \frac{1}{2}r^2 \sin^2 \theta$, into (1) and (2). But the solution so obtained exhibits "wiggles" in the vorticity close to the sphere near $\theta = 0$ and $\theta = \pi$. We attributed these "wiggles" to the fact that the finite-element interpolation of $\sin^2 \theta$ is not very smooth, and to the exacerbating effect of the $1/\sin \theta$ terms in (1) and (2). To get round this difficulty, we replaced the analytical solution for $\psi_{\rm FS}$ by a finite-element approximation, obtained by solving equation (1) with $\zeta = 0$ and appropriate

Dirichlet conditions at $r=1$ and $r=r_\infty$. Whilst the values of this finite-element approximation are not exact on the nodes of the elements near the sphere, it does possess a continuous first derivative, and so the problems with the vorticity are avoided.

An important quantity, which we consider here, is the drag coefficient C_D. This is related to the drag D on the sphere by

$$C_D = 8D/\rho U^2 \pi d^2. \tag{5}$$

It is made up of two contributions: a viscous component C_V and a pressure or "form drag" component C_P. C_V and C_P are given by

$$C_V = -\frac{8}{\text{Re}} \int_0^\pi \{\zeta \sin^2\theta\}_{r=1}\, d\theta, \tag{6}$$

$$C_P = \frac{4}{\text{Re}} \int_0^\pi \left\{\left(\zeta + \frac{\partial \zeta}{\partial r}\right) \sin^2\theta\right\}_{r=1} d\theta. \tag{7}$$

3. Velocity–Pressure Formulation

The other formulation of the Navier–Stokes equations is based on the velocity $\mathbf{u}$ and the pressure p. The non-dimensional form of the equations is

$$\frac{\text{Re}}{2}\, \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u}, \tag{8}$$

$$\nabla \cdot \mathbf{u} = 0. \tag{9}$$

The boundary condition on the surface of the sphere is $\mathbf{u}=0$. We also apply boundary conditions on a spherical surface far from the sphere ($r=r_\infty$). Over the part of the boundary on which the free stream flows into the computational region ($\pi/2 \leqslant \theta \leqslant \pi$) we set the velocity equal to the free stream value, and over the remainder the viscous stress is set to zero. This is a weaker requirement than setting the velocity equal to the free stream value all over the outer boundary, and so we expect to be able to apply it closer to the sphere. However, it does not have the physical basis of zero normal derivative condition (4) for the stream-function–vorticity formulation.

The equations are again solved in spherical polar coordinates using a Galerkin finite-element method. Nine-node rectangular elements, having biquadratic interpolation, were used for the two components of the velocity and piecewise linear interpolation was used for the pressure field. This pressure interpolation is, in general, discontinuous across element boundaries. This implementation in spherical polar coordinates is a trivial extension to the method advocated by Engleman, Sani, Gresho, and Bercovier [15]. From the solutions the viscous and pressure components of the drag coefficient are calculated, but now using

$$C_V = \frac{8}{\text{Re}} \int_0^{\pi} \left\{ -2 \frac{\partial u_r}{\partial r} \sin\theta \cos\theta + \frac{\partial u_\theta}{\partial r} \sin^2\theta \right\}_{r=1} d\theta, \tag{10}$$

$$C_P = \frac{8}{\text{Re}} \int_0^{\pi} \{p \sin\theta \cos\theta\}_{r=1} \, d\theta. \tag{11}$$

4. Comparison of Different Methods

Predictions of the three finite-element methods are examined at two values of Re, 1 and 40, which are typical of low and intermediate Reynolds numbers. Various uniform grids were used, specified by 3 quantities: n_θ the number of elements in the θ direction, n_ξ the number of elements in the ξ direction and ξ_∞ the position of the outer boundary. In the comparison, we examine the best value of C_D that can be obtained for given ξ_∞, the accuracy of each of the three methods for given n_ξ and n_θ, and the most economical calculation for a given accuracy.

A number of values of ξ_∞, the position of the outer boundary, are considered, ranging from $\xi_\infty = 2.4$ ($r_\infty = 11.0$) to $\xi_\infty = 4.8$ ($r_\infty = 121.5$). For the above Reynolds numbers we found that $n_\theta = 20$ and $n_\theta = 30$ gave the same results to within the required accuracy. So the $n_\theta = 20$ solution is regarded as being independent of the number of angular elements. However, for higher Re more elements are needed to resolve the narrower wake. It is not so easy to achieve results independent of the number of radial elements. As C_V in the $\psi - \zeta$ formulation (6) only depends on ζ, it converges as $O(h_\xi^3)$, where h_ξ is the element size in the radial direction (ξ_∞ / n_ξ). However, C_P (7) involves derivatives of ζ and so it converges more slowly as $O(h_\xi^2)$. In the $\mathbf{u} - p$ formulation C_V (10) involves derivatives of the velocity and C_P (11) involves p, and so both converge as $O(h_\xi^2)$. So the results from different grids are used to calculate an extrapolated drag coefficient for each ξ_∞, using an expression of the form

$$v(h_\xi) = e + \alpha h_\xi^2 + \beta h_\xi^3. \tag{12}$$

Three grids, typically with $h_\xi = 0.086$, 0.06, and 0.043 are used to obtain e, α, and β.

Details of the comparison are outlined in Table I. The extrapolated valued of the drag coefficient is given for each ξ_∞. As expected, it is seen that the $\psi - \zeta$ formulation with the zero normal derivative boudary condition (4) gives the most accurate answers, and as a consequence can be applied closer to the sphere. However, for Re = 40 the $\mathbf{u} - p$ formulation with the free stream condition applied upstream is nearly as good. For given ξ_∞, the $\psi - \zeta$ formulation with the free stream condition (3) has the largest error. However, the error would be even larger for a free stream condition over all the outer boundary with the $\mathbf{u} - p$ formulation as the condition on the vorticity is less restrictive than that of zero tangential velocity. For all the methods the boundary has to be further from the sphere for given accuracy for $\mathrm{Re}=1$ than for $\mathrm{Re}=40$, reflecting the fact that the vorticity diffuses further at lower Re.

In addition in Table I for each ξ_∞ the percentage error is shown for two grids: one with $n_\xi = 50$ and the other with $h_\xi = \xi_\infty / n_\xi = 0.06$ (these are the same for $\xi_\infty = 3$). The error is expressed as a percentage of the extrapolated C_D for that ξ_∞, rather than the value as $\xi_\infty \to \infty$. The behaviour for both $\psi - \zeta$ methods is very systematic. The error is approximately constant for given h_ξ as ξ_∞ is increased. As a result the error increases for fixed n_ξ, as the grid becomes coarser. The error is larger for $\mathrm{Re}=40$, than for $\mathrm{Re}=1$. The results for the $\mathbf{u}-p$ formulation are far less systematic. For the smaller values of ξ_∞, when the presence of the outer boundary is affecting the drag significantly, the error is quite large. However, as ξ_∞ is increased and the limiting value of C_D is approached, the error for fixed h_ξ plummets. Remarkably for the values of ξ_∞ considered, even the error for fixed n_ξ falls, so for given n_θ and n_ξ an answer closer to the actual drag coefficient is obtained with a larger ξ_∞ and coarser grid. Thus for the larger values of ξ_∞, the error for given ξ_∞ is significantly lower for the $\mathbf{u}-p$ formulation than for those using ψ and ζ.

It is difficult to see why this should be the case, but the following argument may provide an explanation. The finite-element representation we are using in the $\mathbf{u}-p$ formulation is designed to make the error in the stress tensor stationary (this is only strictly true in Stokes flow). Since the drag is an integral of this stress tensor, we might expect our $\mathbf{u}-p$ formulation to produce more accurate answers than the $\psi - \zeta$ method, for which the error in the stress is not stationary.

In the $\mathbf{u}-p$ formulation more equations are being solved, and on the same grid we estimate that 40 % more computing time is required than for the other two methods. So for calculations restricted to the same computing time, 40 % more elements can be used in the radial direction with the $\psi - \zeta$ formulation, and with quadratic convergence any errors will be approximately halved. However, we see from Table I that provided ξ_∞ is sufficiently large, the $\mathbf{u}-p$ formulation still gives a more accurate value. For $\mathrm{Re}=1$ the difference is fairly small, but for $\mathrm{Re}=40$ it is more appreciable: at $\xi_\infty = 3.6$ the percentage error is 0.08 for the $\mathbf{u}-p$ formulation with $h_\xi = 0.06$; at the same cost but with the $\psi - \zeta$ formulation we could compute with n_ξ increased by 40 %, reducing h_ξ to 0.043, so that the error obtained with the better boundary condition should decrease from 0.58 to about 0.3.

This discussion has focussed on the drag coefficient. If the details of the flow are required, such as the form of the perturbation stream function far from the sphere, the zero normal derivative boundary condition (4) with the $\psi - \xi$ formulation is the only one with a secure physical base.

TABLE I

A Comparison for the Three Solution Methods of the Limiting Value of the Drag Coefficient $C_D(\xi_\infty)(n_\xi, n_\theta \to \infty)$ and the Error on Two Specific Grids for Various Positions of the Outer Boundary (ξ_∞)

r_∞	ξ_∞	Limiting $C_D(\xi_\infty)$ $(n_\xi, n_\theta \to \infty)$	% Error on specific grid $n_\xi = 50$	$\xi_\infty / n_\xi = 0.06$
		(a) Re = 1		
(i) $\psi - \zeta$ formulation, zero normal derivative boundary condition				
20.1	3.0	27.35	0.07	0.07
36.6	3.6	27.33	0.12	0.08
66.7	4.2	27.32	0.14	0.07
121.5	4.8	27.32	0.19	0.07
(ii) $\psi - \zeta$ formulation, free stream boundary condition				
20.1	3.0	28.02	0.08	0.08
36.6	3.6	27.54	0.10	0.07
66.7	4.2	27.38	0.14	0.07
121.5	4.8	27.34	0.19	0.07
(iii) $\mathbf{u} - p$ formulation				
20.1	3.0	27.61	0.38	0.38
36.6	3.6	27.40	0.09	0.07
66.7	4.2	27.34	0.06	0.03
121.5	4.8	27.32	0.05	0.03
		(b) Re = 40		
(i) $\psi - \zeta$ formulation, zero normal derivative boundary condition				
11.0	2.4	1.786	0.40	0.59
20.1	3.0	1.788	0.58	0.58
36.6	3.6	1.789	0.78	0.58
66.7	4.2	1.789	0.98	0.58
(ii) $\psi - \zeta$ formulation, free stream boundary condition				
11.0	2.4	1.820	0.40	0.59
20.1	3.0	1.796	0.60	0.60
36.6	3.6	1.791	0.78	0.58
66.7	4.2	1.789	0.98	0.58
(iii) $\mathbf{u} - p$ formulation				
11.0	2.4	1.777	0.46	0.63
20.1	3.0	1.788	0.14	0.14
36.6	3.6	1.789	0.10	0.08
66.7	4.2	1.789	0.08	0.05

Note. The error is a percentage of $C_D(\xi_\infty)$ rather than the value as $\xi_\infty \to \infty$.

TABLE II

The Computed Drag Coefficient C_D, with the Pressure and Viscous Components C_P and C_V Compared with Previous Calculations of C_D [1–5] and with the Correlations (13) ($Re \leqslant 20$) and (14) ($Re \geqslant 20$) of Clift *et al.* [9]

		Le Clair *et al.* [2][a]					Correlation of	Present calculations		
Reynolds Number	Hamielec *et al.* [1]	Value from best grid	Corrected value	Dennis and Walker [3][b]	Sayegh and Gauvin [4]	Renksizbulut [5]	Clift *et al.* [9]	C_D	C_P	C_V
1	27.5	27.375	27.315	27.44	27.49	—	27.16	27.32	9.13	18.19
2	—	—	—	—	—	—	14.96	14.92	5.01	9.90
5	—	7.121	7.029	7.210	—	—	7.033	7.138	2.447	4.691
10	—	4.337	4.288	4.424	4.59	4.547	4.259	4.308	1.527	2.781
15	—	—	—	—	—	—	3.253	3.274	1.195	2.079
20	2.78	2.736	2.711	2.730	2.86	2.851	2.715 2.735	2.719	1.019	1.700
30	—	2.126	2.110	—	2.21	—	2.122	2.118	0.830	1.288
40	1.86	—	—	1.808	—	1.728	1.788	1.789	0.728	1.061
50	—	—	—	—	1.62	—	1.574	1.576	0.661	0.915
70	—	—	—	—	—	1.304	1.309	1.311	0.580	0.731
80	—	—	—	—	—	—	1.220	1.222	0.553	0.669
100	1.12	1.096	—	—	—	1.069	1.087	1.088	0.512	0.576

[a] Two values of Le Clair *et al.* are quoted, the first is for the finest, most extensive grid, and the second has a factor applied to correct for grid-size effects and the position of the outer boundary.

[b] These values are twice those quoted by Dennis and Walkter, as a different definition of C_D is used.

5. Comparison with Earlier Work

Finally in Table II we compare the results of our calculations of C_D for Reynolds numbers in the range 1 to 100 with previous calculations [1–5]. Our computed values of C_P and C_V are also given. In addition the values of C_D are compared with the correlations developed by Clift *et al.* [9], based on earlier work. They suggest correlations covering all values of Re; and the two expressions relevant to the present results are

$$C_D = \frac{24}{\mathrm{Re}} \left[1 + 0.1315\,\mathrm{Re}^{(0.82 - 0.05\log_{10}\mathrm{Re})}\right] \qquad 0.01 \leqslant \mathrm{Re} \leqslant 20, \tag{13}$$

$$C_D = \frac{24}{\mathrm{Re}} \left[1 + 0.1935\,\mathrm{Re}^{0.6305}\right] \qquad 20 \leqslant \mathrm{Re} \leqslant 260. \tag{14}$$

Our results quoted in Table II were those obtained using the $\psi - \zeta$ formulation and the zero normal derivative boundary condition (4); and we believe the values of C_D are accurate to within 2 in the fourth significant figure. We see that the correlation (14) is rather better for intermediate Reynolds number than (13) is for low Reynolds numbers, where there are errors of up to 1.5 %.

6. Conclusions

The zero normal derivative condition (4) used with the $\psi - \zeta$ formulation and imposed on the boundary of the solution region far from the sphere is the only one of the conditions we have considered with a secure physical basis, and it can be applied closer to the sphere than the other two. In addition, it is well suited for computing the overall flow. However, we have found that, as long as ξ_∞ is sufficiently large for the extrapolated value of C_D for that ξ_∞ to be an accurate estimate of the actual value of C_D, then coarser grids can be used with the $\mathbf{u} - p$ formulation than with the $\psi - \zeta$ formulations for a given acceptable error. Indeed, even though for the same grid the $\mathbf{u} - p$ formulation is approximately 40 % more expensive, this is more than compensated for by the coarser grid, so for a given required accuracy in the drag coefficient, the $\mathbf{u} - p$ formulation is cheaper.

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REFERENCES

1. A. E. HAMIELEC, T. W. HOFFMAN, AND L. L. ROSS, *AIChE J.* **13** (1967), 212.
2. B. P. LE CLAIR, A. E. HAMIELEC, AND H. R. PRUPPACHER, *J. Atmos. Sci.* **27** (1970), 308.
3. S. C. R. DENNIS AND J. D. A. WALKER, *J. Fluid Mech.* **48** (1971), 771.
4. N. N. SAYEGH AND W. H. GAUVIN, *AIChE J.* **25** (1979), 522.
5. M. RENKSIZBULUT, "Energetics and Dynamics of Droplet Evaporation in High Temperature Intermediate Reynolds Number Flows," Ph.D. thesis, Nortwestern University, Evanston, Ill., 1981.
6. M. RENKSIZBULUT AND M. C. YUEN, *Am. Soc. Mech. Eng. Heat Transfer Div.* **105** (1983), 389.
7. K. A. CLIFFE AND D. A. LEVER, *in* "Proc. 3rd Int. Conf. on Numerical Methods in Laminar and Turbulent Flow" (C. Taylor, J. A. Johnson, and W. R. Smith, Eds.), p. 25, Pineridge, Swansea, 1983.
8. K. A. CLIFFE AND D. A. LEVER, "A Finite-Element Study of Isothermal, Laminar Flow Past a Sphere at Low and Intermediate Reynolds Numbers," Harwell Report AERE-R.10868, H.M.S.O., 1983.
9. R. CLIFT, J. R. GRACE, AND M. E. WEBER, "Bubbles, Drops and Particles," Academic Press, New York, 1978.
10. G. K. BATCHELOR, "An Introduction to Fluid Dynamics," Cambridge Univ. Press, New York/London, 1970.
11. B. FORNBERG, *J. Fluid Mech.* **98** (1980), 819.
12. P. TONG, *in* "Proc. 1st Int. Conf. on Finite-Element Methods in Flow Problems," Swansea, 1974, p. 57.
13. K. H. WINTERS AND K. A. CLIFFE, "A Finite-Element Study of Driven Laminar Flow in a Square Cavity," Harwell Report AERE-R.9444, H.M.S.O., 1982.
14. C. P. JACKSON, "The TGSL Finite-Element Subroutine Library," Harwell Report AERE-R.10713, H.M.S.O., 1982.
15. M. S. ENGLEMAN, R. L. SANI, P. M. GRESHO, AND M. BERCOVIER, *Int. J. Numer. Meth. Fluids* **2** (1982), 25.